

## Numerical Modelling of Tidal Circulation and Sediment Transport in the Gulf of Khambhat and Narmada Estuary, West Coast of India

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### ABSTRACT

A depth-averaged numerical model was developed to study tidal circulation and suspended sediment transport in the gulf of Khambhat along the west coast of India. The spatial resolution of the model is 750m x 750m. A 2-D fine resolution (150 m x 150 m) model for the lower part of the Narmada estuary is coupled with the coarser gulf model to simulate the flow features in the lower estuary. The model dynamics and basic formulation remain the same for both the gulf model and the estuary model. The models are barotropic, based on the shallow water equations and neglect horizontal diffusion and wind stress terms in the momentum equations. The models are fully non-linear and use a semi-explicit finite difference scheme to solve mass, momentum, and advection-diffusion equation for suspended sediments in a horizontal plane. The erosion and deposition have been computed by an empirically developed source and sink term in the suspended sediment equation. The tide in the gulf is mainly represented in the model by the semi-diurnal  $M_2$  constituent. Meanwhile, fresh water discharge from the rivers joining the gulf had also been considered. Numerical experiments were carried out to study the circulation and suspended sediment concentrations in the gulf and estuarine region.

**Keywords:** Tidal circulation, suspended sediment, numerical model,  $M_2$  constituent, gulf of Khambhat, Narmada estuary

### INTRODUCTION

In the recent years, the importance of sediment transport to marine environment and coastal engineering has been widely recognized. Johns *et al.* (1990) used a numerical model to determine changes in bed morphology resulting from bed load and suspended transport of sand in the Taw estuary, England. Podsetchine and Hutulla (1994) developed a 2-D vertical flow and suspended matter transport model for Lake Karhijarvi, Finland. A coupled hydrodynamic/suspended sediment transport model, including waves and currents, was developed by Lou and Ridd (1997). The model system has been applied to Cleveland Bay, Australia. Wu *et al.* (2000) used a numerical model for calculating flow and sediment transport in open channels. In their model, the water level is determined from a 2-D Poisson equation derived from depth-averaged momentum equations. Suspended load transport is simulated through the general advection-diffusion equation with an empirical settling velocity term. Tattersall *et al.* (2003) applied 2-D depth-averaged models to simulate the tidal currents and suspended sediment concentrations in the Tamar estuary located in the south west coast of England. Recently, Gleizon *et al.* (2003) developed a model of vertically resolving sediments

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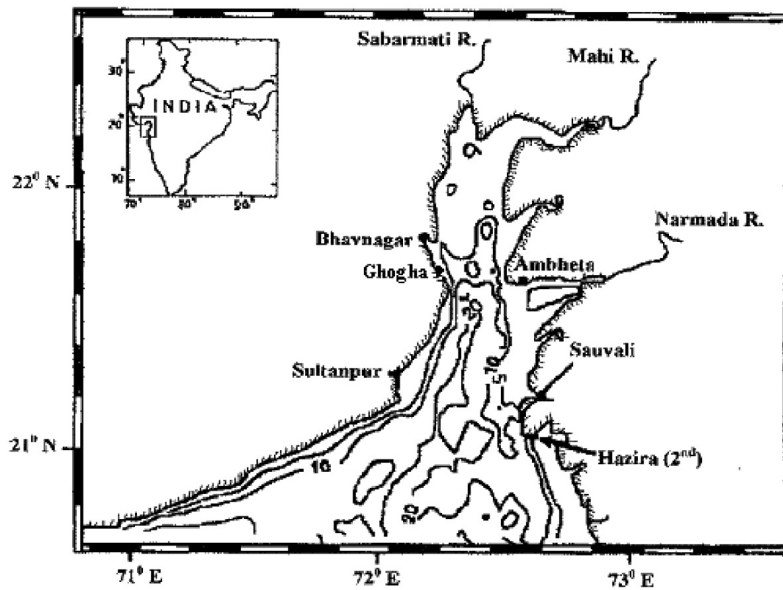


Fig. 1: Map of the gulf of Khambhat

in estuaries (VERSE) to simulate water, sediment, and sediment-bound contaminant transport in vertically stratified and relatively narrow microtidal estuary (Ribble estuary).

The gulf of Khambhat (Fig. 1) is an inverted funnel-shaped indentation on the western shelf of India, between the Saurashtra Peninsula and the mainland of Gujarat. The north-south extent of the analysis region is approximately 145 km, while its width varies between 20 and 110 km. The gulf has an area of about 3120 sq. km and a maximum depth of 35 m. The important characteristic of the gulf is its high tidal range. In the gulf, tidal currents dominate the flow. The tides are of semi-diurnal type with a large diurnal inequality and varying amplitudes, which increase from the south to north along the gulf coast. The height of the tide increases tremendously from the mouth to the upstream end because of the funnel-shape of the gulf and the semi-enclosed nature at the head and resonance. Meanwhile, tidal currents are fairly strong and bimodal in nature with two dominant directions; towards upstream during flood and downstream during ebb in all encompassing oscillatory motions. The maximum currents occur during mid-tide, which is around 2.5 m/s in the gulf, and associated with high wave energy (Sen Gupta and Deshmukhe, 2000). The gulf is more or less homogeneous displaying a one layer structure. This is caused by the shallowness of the depths and medium to high tidal amplitudes associated with tidal currents and turbulence.

The rivers draining into the gulf of Khambhat carry enormous amount of sediments in their discharges. It receives drainage from Sabarmati, Mahi, and Narmada rivers. These rivers discharge a large volume of sediments as also the suspended load. The bottom consists of mainly the river-borne fine- to coarse-grained sand. The western and northern parts of the gulf consist of largely soft sediments of Quaternary rocks. On the eastern side of the gulf, the river Narmada carries a large volume of suspended sediments. At the estuarine mouth of the river Narmada, the alluvium gets deposited creating deltaic islands like Alia Bet. Keeping this in view, the researchers developed a depth-averaged numerical model to study the tidal flow and suspended sediment transport in the gulf of Khambhat. Nevertheless, the gulf model could not simulate the realistic circulation pattern and suspended sediment concentration near the river mouths. Compared to the other two rivers,

Narmada is a major river in the region. Therefore, a 2-D fine resolution model for the lower part of the Narmada estuary is coupled with the gulf model to simulate the flow features in the lower estuary. The model dynamic and basic formulations remain the same for both the gulf and estuary models. The model equations in the depth-averaged form include non-linear terms and the solution was obtained using a conditionally stable semi-explicit finite difference scheme on a staggered grid.

### FORMULATION OF THE MODEL

In the formulation of the model, two-dimensional vertically integrated shallow water equations are used. Neglecting the sphericity of the earth's surface, a system of rectangular Cartesian coordinate is used in which the origin 'O' is within the equilibrium level of the sea surface. Ox points eastward, Oy points northward and Oz is directed vertically upwards. The displaced position of the free surface is given by  $z = \zeta(x, y, t)$ , and the position of the seal floor by  $z = -h(x, y)$ .

Assuming hydrostatic approximations, the resulting equations of continuity and momentum are given by:

$$\frac{\partial \zeta}{\partial t} + \frac{\partial(Hu)}{\partial x} + \frac{\partial(Hv)}{\partial y} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - fv = -g \frac{\partial \zeta}{\partial x} + A \nabla^2 u + \frac{1}{H} \left[ \frac{F_s}{\rho} - \frac{gu(u^2 + v^2)^{\frac{1}{2}}}{C^2} \right] \quad (2)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + fu = -g \frac{\partial \zeta}{\partial y} + A \nabla^2 v + \frac{1}{H} \left[ \frac{G_s}{\rho} - \frac{gv(u^2 + v^2)^{\frac{1}{2}}}{C^2} \right] \quad (3)$$

where  $H (\zeta + h)$  is the total depth,  $f$  is the Coriolis parameter at 22° N,  $g$  is the acceleration due to gravity, while  $u$  and  $v$  are the  $x$  and  $y$  components of depth averaged velocity, respectively,  $F_s$  and  $G_s$  are the surface shear stress due to wind in the  $x$  and  $y$  directions, and  $A$  is the horizontal eddy viscosity coefficient.

Here, the researchers neglected horizontal diffusion terms in the momentum equations, as their effect was very small as compared to other terms. In addition, the surface stress terms were also neglected as their effect was found to be very small on the tidal circulation in the gulf. The bottom stress terms are parameterized by conventional quadratic laws, and the Chezy coefficient is evaluated by:

$$C = n^{-1} h^{\frac{1}{6}}; \text{ where, } n = 0.025$$

For the sediment transport prediction, a depth-averaged form of the advection-diffusion equation was used to represent the deposition and erosion rates, together with the source and sink terms.

$$\frac{\partial(HC_{ss})}{\partial t} + \left[ \frac{\partial(uHC_{ss})}{\partial x} + \frac{\partial(vHC_{ss})}{\partial y} \right] = \frac{\partial}{\partial x} \left[ HKC \frac{\partial C_{ss}}{\partial x} \right] + \frac{\partial}{\partial y} \left[ HKC \frac{\partial C_{ss}}{\partial y} \right] + \gamma w_s (C_e - C_{ss}) \quad (4)$$

where  $C_{ss}$  is the depth-averaged suspended sediment concentration,  $w_s$  is the particle settling velocity,

$K_c = 150 \text{ m}^2/\text{s}$  is the horizontal diffusivity coefficient of the sediment and  $C_c$  is the depth-averaged equilibrium concentration. The profile factor  $\gamma$  is given as (Johns *et al.*, 1990)

$$\gamma = \frac{C_{ss-h}}{C_{ss}} \tag{5}$$

Numerical experimentation has shown that the concentrations of the sediment are in the observed range for the value of  $\gamma$  close to 1. Hence, the value of  $\gamma$  was taken as 1 in the present study.

In determining the equilibrium sediment concentration, the formula suggested by Engelund and Hansen (1967) was used.

*Boundary Conditions and Methods of Solution*

**Tidal boundary condition**

The gulf has an open boundary at the seaward end connecting  $71.75^\circ \text{E}$  and  $72.64^\circ \text{E}$  at  $21^\circ \text{N}$  (Fig. 2). At the landward end, there are three open boundaries of the analysis area connecting Sabarmati, Mahi, and Narmada rivers. A realistic tidal boundary condition, used at the southern boundary and freshwater discharges from the three rivers, is provided at the river mouths.

The observations of the amplitude and phase of  $M_2$  tide (Unnikrishnan *et al.*, 1999) at the

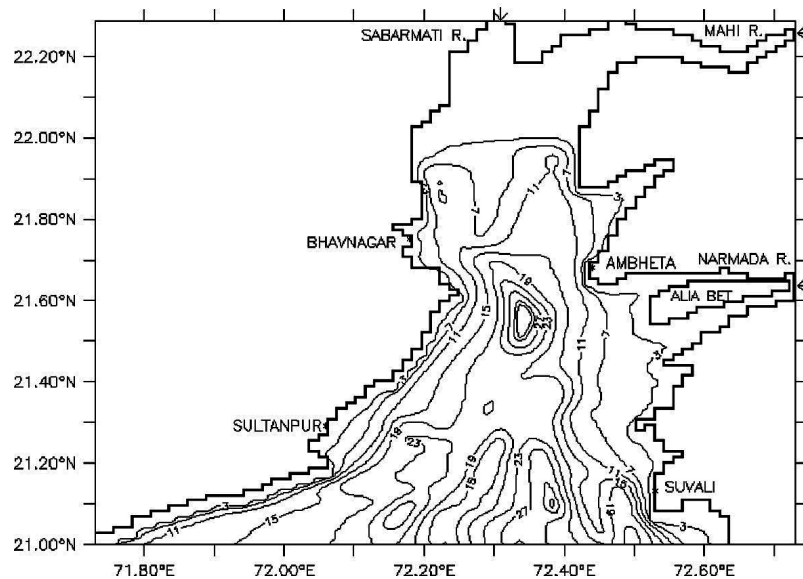


Fig. 2: Bathymetry (m) in the gulf of Khambhat

stations located at the extreme left and extreme right ends of the southern boundary are given in the form

$$\zeta = 1.0 \cos (\omega t - 80^\circ) \text{ and } \zeta = 1.75 \cos (\omega t - 90^\circ) \tag{6}$$

Using these values, the amplitude and phase at the remaining grid points of the open boundary are derived from a linear interpolation.

At the river mouth, the fresh water discharge was prescribed based on available observations. The average runoff into the gulf from Narmada river is nearly 800 m<sup>3</sup>/s (Haskoning Consulting Engineers and Architects, 1990), while Sabarmati and Mahi contribute about 400 m<sup>3</sup>/s each. Fresh water discharge from the river mouth is given in the following form

$$\bar{u} = \frac{Q}{A} \quad (7)$$

where Q is the discharge at a river mouth and A is the cross sectional area at that point.

### Sediment boundary condition

The sinusoidally varying suspended sediment concentrations along the southern boundary were calculated using the observed values at the extreme ends of the boundary, which are given by:

$$C_{SS} = 200 \cos(\omega t - 170^\circ) \text{ and } C_{ss} = 350 \cos(\omega t - 180^\circ) \quad (8)$$

Along the mouth of the rivers, the sediment concentrations are given by:

$$\begin{aligned} C_{ss} &= 260 \cos(\omega t - 270^\circ) \text{ at Sabarmati river mouth} \\ C_{ss} &= 200 \cos(\omega t - 273^\circ) \text{ at Mahi river mouth} \\ C_{ss} &= 520 \cos(\omega t - 214^\circ) \text{ at Narmada river mouth} \end{aligned} \quad (9)$$

where amplitude is in mg/l,  $\omega = \frac{2\pi}{T}$  and T is the tidal time period.

In addition to the no-slip bottom boundary condition, appropriate conditions have to be satisfied along the lateral boundaries of the gulf area under consideration at all times. In the vertically integrated system at the water land interface, the transport normal to coastline and sediment diffusive fluxes are zero. The motion in the gulf is generated from the initial state of rest and constant sediment concentration. Thus, starting from these initial conditions, an oscillatory steady state was achieved after the 8<sup>th</sup> tidal cycle of integration, while the results of the 9<sup>th</sup> tidal cycle were also analysed.

The equations of continuity and momentum were solved using a semi-explicit finite difference scheme on Arakawa-C grid (Sinha *et al.*, 1996), while the suspended sediment transport equation was solved using an explicit finite difference method with advective terms which were represented by an upstream finite difference method.

## NUMERICAL EXPERIMENTATION

The analysis areas were chosen from 21° N to 22° 18' N and 71° 45' E to 72° 43' E (*Fig. 1*) to study the tidal circulation and the suspended sediment transport in the gulf of Khambhat. Meanwhile, the coastline of the gulf which includes the island of Alia Bet is made up of stair steps consisting of straight line segments parallel to x- and y-axes (*Fig. 2*). In this study, the east-west extent of the gulf is 112.50 km and the north-south extent is 143.25 km. A rectangular mesh of (151 x 192) grid points was chosen to give  $\Delta x = \Delta y = 750$  m. In order to avoid sharp depth gradients, a smooth bathymetry was used and the depths in the gulf varied from 3 m to 32 m. The bathymetry in the gulf region is quite irregular and there are many shallow zones followed by deep channels on either side (*Fig. 2*).

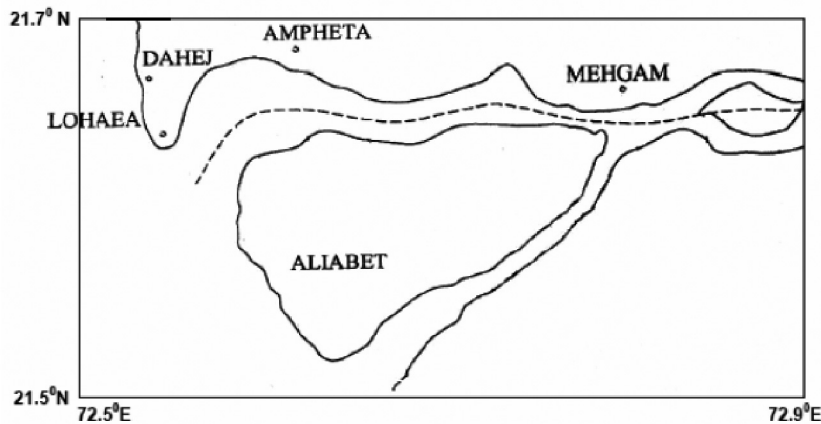


Fig. 3: Narmada river estuary

Compared to the other two rivers, Narmada is a major river in this region. The fresh water discharge from the Narmada estuary is nearly  $800\text{m}^3/\text{s}$ , and it also contributes a higher amount of suspended sediments in the gulf. The coarser gulf model could not compute the tidal flow and sediment transport in the lower part of the Narmada estuary. Hence, a fine resolution model was developed for the lower Narmada estuary, coupled with the coarser gulf model, to compute the flow features in the estuarine region. The gulf and estuary models are dynamically linked together at the interface through the elevation points. The tidal amplitudes and sediment concentrations computed from the coarser model are provided as input to the estuary model.

In order to study the flow features in the lower part of the Narmada river estuary (Fig. 3), the estuarine area from Lohaea to Mehgam was chosen. The stair-step boundaries were used to represent the island as well as the irregular coastal boundary of the estuary. Here, the length was taken as 16.65 km, while the breadth as 28.5 km. The area was represented using a mesh of  $191 \times 112$  grid points with a grid interval of 150 m in both x- and y- directions and a constant depth of 6 m was chosen. The time step of the fine resolution estuary model is taken as 10 seconds.

## RESULTS AND DISCUSSION

The basic equations, along with appropriate boundary and initial conditions, have been integrated ahead of time till a steady state is reached. During flood, the computed  $M_2$ -tidal amplitudes at Suvali and Ambheta were 1.93 m and 2.58 m, respectively. Table 1 provides a comparison of the observed and computed amplitudes at the two stations in the gulf. The computed tidal amplitudes were found to be in agreement with the available observations (Unnikrishnan *et al.*, 1999).

The model is able to be used to produce the flood-ebb asymmetry in the gulf region. Figs. 4a and 5a show the computed depth-averaged currents after 3 hours (flood) and 9 hours (ebb) of the 9<sup>th</sup> tidal cycle, respectively. In the gulf, the currents during flood vary from 0.1 m/s to 2.0 m/s. The tidal currents during ebb are found to vary from 0.1 m/s to 2.21 m/s. It was found that the currents are generally stronger in the central portion of the gulf.

As indicated previously, the coarser gulf model could not simulate the tidal flow near the mouth of the Narmada estuary. Therefore, a fine resolution model for the lower part of the estuary was combined with the coarser gulf model to compute the flow features in this particular estuarine region. Figs. 4b and 5b depict the computed flood and ebb currents in the Narmada river estuary. It was observed that the computed depth-averaged flood currents varied from 0.01 m/s to 1.2 m/s,

TABLE 1  
Observed and computed amplitudes (m) of  $M_2$  tide during flood at two stations in the gulf of Khambhat

Station	Observed amplitude	Computed amplitude
Suvali	2.05	1.93
Ambheta	2.07	2.58

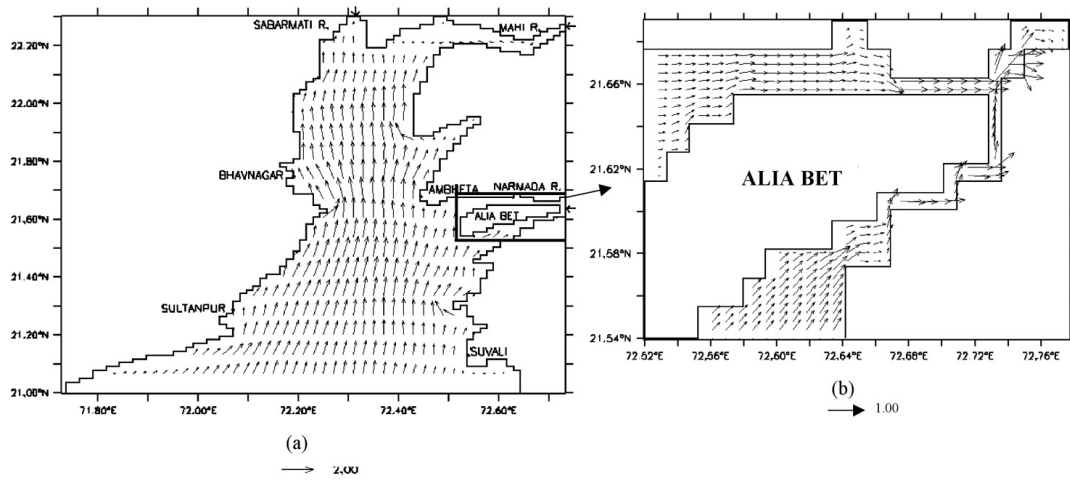


Fig. 4: Tidal currents (m/s) in (a) the gulf of Khambhat; (b) the lower part of the Narmada estuary during flood

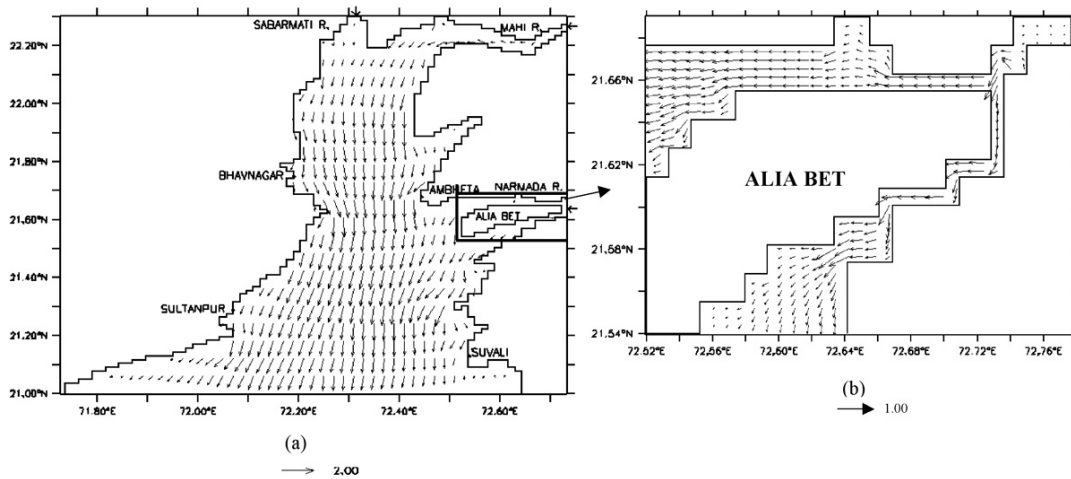


Fig. 5: Tidal currents (m/s) in (a) the gulf of Khambhat; (b) the lower part of the Narmada estuary during ebb



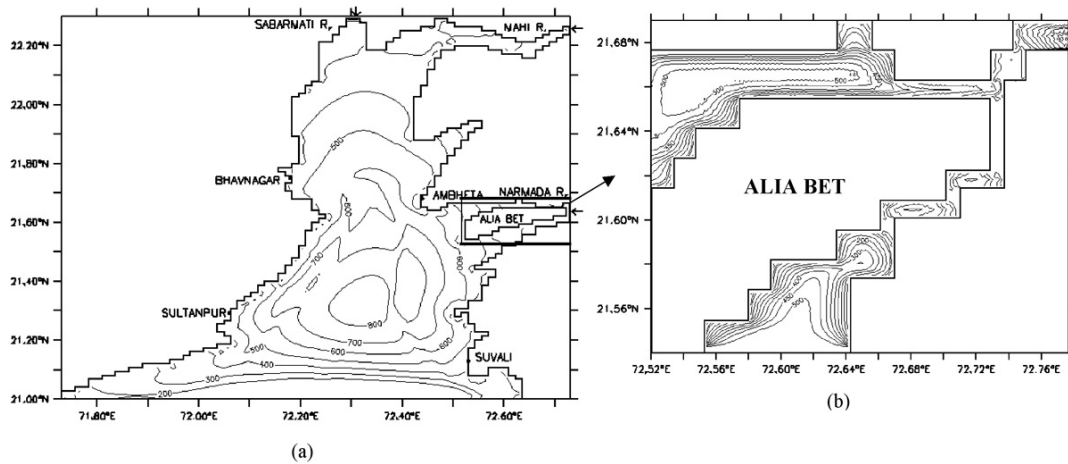


Fig. 6: Suspended sediment concentrations (mg/l) in (a) the gulf of Khambhat; (b) the lower part of the Narmada estuary during flood

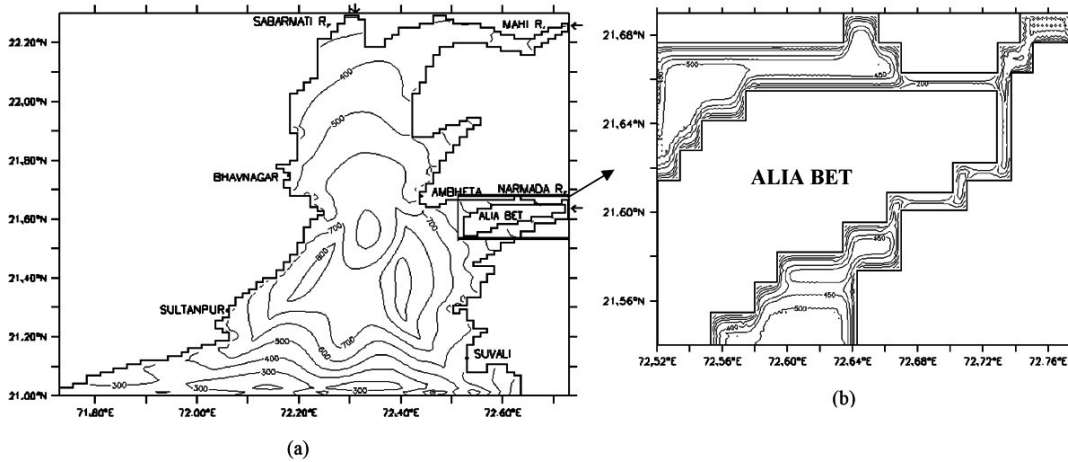


Fig. 7: Suspended sediment concentrations (mg/l) in (a) the gulf of Khambhat; (b) the lower part of the Narmada estuary during ebb

while during ebb it varied from 0.01 m/s to 1.0 m/s. The flow bifurcation near Alia Bet Island was well reproduced in the fine resolution estuary model.

The computed suspended sediment concentrations in the gulf region during flood and ebb periods are shown in Figs. 6a and 7a, respectively. During flood, the sediment concentrations varied from 100 mg/l to 800 mg/l and it varied from 120 mg/l to 920 mg/l during ebb. As expected, the concentrations were higher during the ebb than during the flooding period. This is because of the joining of the three rivers, namely Sabarmati, Mahi and Narmada, in the gulf.

Figs. 6b and 7b show the computed suspended sediments in the Narmada estuary during flood and ebb periods, respectively. The lower part of the estuary is divided into two parts due to the



TABLE 2  
Observed and computed mean suspended sediment concentrations (mg/l) in the Narmada river estuary

Year (Period)	Observed SSC*	Computed SSC*
1977 (Post-monsoon)	540	-----
1978 (Pre-monsoon)	430	515

\*SSC – Suspended Sediment Concentrations

presence of Alia Bet Island. During flood, the sediment concentrations were found to vary from 60 mg/l to 500 mg/l in the upper part and from 40 mg/l to 508 mg/l in the lower part. However, the sediment concentrations varied from 110 mg/l to 517 mg/l in the upper part and from 100 mg/l to 530 mg/l in the lower part during the ebb period. The computed mean of the maximum concentration in the lower part of the estuary was found to be 515 mg/l. The computed sediment concentrations in the lower estuary were found to be in the range of available observations, as shown in Table 2 (Borole *et al.*, 1982).

### CONCLUSIONS

A depth-averaged numerical model was developed to compute the tidal circulation and suspended sediment transport in the gulf of Khambhat by including three rivers, namely, Sabarmati, Mahi and Narmada. A 2-D fine resolution model for the lower part of the Narmada estuary was combined with the gulf model to simulate the flow features in the lower estuary. The computed  $M_2$ -tidal amplitudes, during flood at Suvali and Ambheta, were found to be in reasonable agreement with the available observations. The flow bifurcation near Alia Bet Island was well-reproduced in the fine resolution estuary model. Meanwhile, the computed mean suspended sediments were validated with the available observations in the lower estuary. Reasonable circulation pattern and suspended sediment concentrations have been simulated in the gulf model and the estuary model. However, a better representation of the wind stress terms and the inclusion of horizontal diffusion in the momentum equations would be more appropriate.

### ACKNOWLEDGEMENTS

The authors gratefully acknowledged the Universiti Malaysia Terengganu and the Indian Institute of Technology, Delhi for the facilities provided at the Institute of Oceanography and the Centre for Atmospheric Sciences, respectively.

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